Flame Shape of Buoyant Jet Diffusion Flames at Sub-Atmospheric Pressures

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ABSTRACT

This paper reports an interesting phenomenon about the flame shape of laminar jet diffusion methane flames under sub-atmospheric pressures. Experiments were conducted in a confined cabin with internal size of 3 m × 2 m × 2 m and design pressure of 45-100 kPa. The methane flame was produced by a circular burner with an inner diameter of 12 mm. The fuel mass flow rates varied in the range of 6-21 mg/s, with the jet exit Reynolds numbers of 58-202 and Froude numbers of 0.046-2.8. The dimensionless flame length and radius were analyzed to have the relations of \((l/\tau) (r/\tau)^{4/3} \sim Fr^{1/3}\) and \(l/\tau \sim Re\), which were in good agreement with the experimental results. The relationship of the dimensionless flame length and Froude number as well as that between the dimensionless flame length and Reynolds number were \(l/\tau = 20.2 Fr^{0.322}\) and \(l/\tau = 0.123 Re\), respectively. The flame length normalized as \(l/\tau \sim Re\) increased with the reduction of pressure according to \(l/\tau = 0.12 p_o^{-0.22}\). The oxygen density decreased with the decrease of ambient pressure, and the rate of air needed for combustion was proportional to the stoichiometric requirements for combustion. As a result, the flame sheet at lower ambient pressure moved upward to obtain adequate fresh air. But this increase of flame length was very small related to the scale of the flame length.

KEYWORDS: Dimensionless number, empirical correlations, flame length, sub-atmospheric pressure.

NOMENCLATURE

\(D\) diffusion coefficient \((m^2/s)\)  
\(Fr\) Froude number (-)  
\(g\) gravitational acceleration \((9.81 \text{ m/s}^2)\)  
\(g'\) acceleration due to buoyancy \((\text{m/s}^2)\)  
\(l\) length \((m)\)  
\(m\) mass flow rate \((\text{kg/s or mg/s})\)  
\(p\) pressure \((\text{kPa})\)  
\(r\) radius \((m)\)  
\(Re\) Reynolds number (-)  
\(S\) volume ratio of air to fuel for stoichiometric combustion (-)  
\(T\) temperature \((K)\)  
\(u\) velocity \((\text{m/s})\)  
\(\dot{V}\) volumetric flow rate \((\text{m}^3/s)\)  
\(Y_{F,\text{stoic}}\) stoichiometric fuel mass fraction (-)  
\(\rho\) density \((\text{kg/m}^3)\)  
\(\mu\) dynamic viscosity \((\text{Pa·s})\)  

Greek

\(\rho\) density \((\text{kg/m}^3)\)  

Subscripts

\(\infty\) ambient condition  
\(0\) initial condition  
\(f\) fuel  
\(F\) flame

INTRODUCTION

Laminar jet diffusion flames have been the subject of much fundamental research due to their controllability and stability. Pressure effect on the flame characteristics has been a hot research topic.
for many years. Research studies have been mainly at elevated pressure conditions over 101 kPa focusing on the flame image characteristic or combustion behavior. For sub-atmospheric pressure conditions, some experimental works achieved in cities at high altitude or in low pressure cabins were reported recently. Both show that there is a significant pressure effect on flame length.

As an important parameter in fires, flame length has been modeled successively since Burke and Schuman’s theory was established in 1928 [1]. In established theories of flame shape, many scholars neglected the influence of buoyancy forces on axial gas velocity for circular port burners [2-5]. Roper’s solution, known as one of the most classic solutions of laminar diffusion flame length for circular port burners, was built up with no assumptions on the value of axial velocity,

\[
l_F = \frac{\dot{V}_f}{4\pi D_\infty \ln \left(1 + 1/S\right) \left(\frac{T_\infty}{T_F}\right)^{0.67}},
\]

where \(l_F\) is the flame length (m), \(\dot{V}_f\) is the volumetric flow rate of fuel gas at ambient temperature and pressure (m\(^3\)/s), \(D_\infty\) is the diffusion coefficient at temperature \(T_\infty\) (m\(^2\)/s), \(S\) is the volume ratio of air to fuel for stoichiometric combustion, the subscripts of \(\infty\), \(f\), \(F\) are the ambient condition, fuel, and flame, respectively. Eq.(1) implied that buoyancy force or secondary air velocity had no effects on flame height, i.e., ambient pressure had no effect on flame height [4].

However, the effect of buoyancy on gas velocity at the flame axis in actual flames is notable. Results that contrast with previous findings have been reported [6-8]. For instance, Yuan et al. [6] modeled the diffusion flame based on a cylindrical flame geometry to predict the trend of flame length variation for steady flames, which considered the buoyancy effects. For steady flames, the flame length increased slightly or remained nearly constant with the reduction of the pressure; while it decreased for flickering flames. Hu et al. [8] found that the dimensionless mean flame height for turbulent jet flames was higher at a lower pressure than that at normal pressure.

The present study has focused on the influence of sub-atmospheric pressure on the flame length and width. Furthermore, incorporating the flame length and width with dimensionless numbers can benefit the research on laminar flame shape significantly, which interprets the flame dynamics physically. The relationships of dimensionless flame length and width, and the jet Froude number and Reynolds number were deduced.

THEORETICAL METHODS

Flame length with Reynolds numbers

In order to develop an understanding of the basic flow and diffusional processes that occur in laminar jet flames, Turner [9] considered a simple case of a non-reacting laminar jet of fuel flowing into infinite, quiescent air. By assuming that there is no source or sink of fluid along the jet axis, the flow is axisymmetric, and at large radii the fluid is stagnant and no fuel is present, the flame length expression was obtained by solving the boundary-layer equations [9],

\[
l_F = \frac{0.375Re \cdot r}{Y_{f,stoic}},
\]

where \(Y_{f,stoic}\) is the stoichiometric fuel mass fraction (-). This expression indicates that the dimensionless flame length is proportional to the jet Reynolds number, just as the conclusion of Eq. (1). In addition, numerous experimental investigations [10-12] revealed that too, as

\[
l_F/d \sim Re.
\]
For a given fuel and oxidizer, the relationship indicates that $l_F/d$ is proportional to the Reynolds number and independent of ambient pressure and burner diameter, which is an interesting prediction for sub-atmospheric pressure conditions in the present setting.

**Flame length and radius with Froude numbers**

Based on Roper’s solution, Wang et al. [13] deduced the flame radius $r_F$ equation for a laminar jet flame produced by a circular port as

$$r_F = \left[ \frac{2 \ln(1+1/S) \dot{m}_f}{\pi g'} \frac{D_{\infty}}{\rho_f} \right]^{1/4} \left( \frac{T_F}{T_{\infty}} \right)^{2/3},$$

where $\dot{m}_f$ is the fuel mass flow rate (kg/s), $\rho$ is the density (kg/m$^3$), $g'$ is the acceleration due to buoyancy ($= (T_F/T_{\infty} - 1)g$), and $g$ is the gravitational acceleration (9.81 m/s$^2$). For a given fuel mass flow rate, the flame temperature is almost independent of ambient pressure, so $g'$ is invariant. Thus, considering $\rho_f \sim \rho_{\infty}$ and $D_{\infty} \sim \rho_{\infty}^{-1}$, the dependence of $r_F$ on ambient pressure is derived as

$$r_F \sim \rho_{\infty}^{-1/2}.$$  

From Eq. (5) and assuming the flame length to be independent of ambient pressure, for a given burner and fuel mass flux, one obtains

$$\frac{l_F}{r} \left( \frac{r_F}{r} \right)^{4/3} \sim \rho_{\infty}^{\alpha} \left( \rho_{\infty}^{1/2} \right)^{4/3} \sim \rho_{\infty}^{-1/3}.$$  

where, $r$ is the radius of the burner (m). Furthermore, the Reynolds number $Re = \rho_f u_{f,\infty} d / \mu_f \sim \rho_{\infty}^0$ and the Froude number $Fr = u_{f,\infty}^2/(gd) \sim \rho_{\infty}^{-2}$, so

$$\frac{l_F}{r} \left( \frac{r_F}{r} \right)^{4/3} \sim Fr^{1/3}.$$  

This result is consistent with the theory of Altenkirch et al. [14], which used the thickness of a natural-convection buoyancy layer on an isothermal vertical flat plate as an approximation of the diffusion flame height.

**EXPERIMENTAL METHODS**

Experiments were conducted in a confined cabin with internal size of 3 m × 2 m × 2 m and design pressure of 45-100 kPa. Fig. 1 shows the diagram of the experimental setup. More details about the experimental setup were reported previously in [13, 15].

![Figure 1. Diagram of the experimental setup.](image-url)
The methane flame was produced by a circular burner with inner diameter of 12 mm. The burner was at the center of the cabin, and the tip of the burner was approximately 0.50 m above the floor of the cabin to locate any flame outside the boundary layers of the cabin walls. The fuel with a purity of 99.99% was supplied by a compressed gas cylinder. The methane mass flow rates were designed at 6.0, 9.0, 12.0, 14.9, 17.9, and 20.9 mg/s, with the jet exit Reynolds numbers of 58-202 and Froude numbers of 0.046-2.8. The jet flow with these flow rates is in the laminar flow range, and the flame is tractable and stable for experimental measurement. A calibrated mass flow meter with precision of ±(0.8% of reading + 0.2% of full scale) was utilized to measure and control the fuel flow.

A CCD (Charge Coupled Device) camera with resolution of 1280 × 720 at 25 frames per second, was used to record the flame images. For an oscillating flame, the flame height is defined as the vertical distance above the nozzle exit surface where the occurrence probability is 0.5; for a stable flame, the flame length is defined as the vertical distance between the nozzle exit surface and the tip of the luminous flame sheet [13]. B-type thermocouples (diameter of 0.1 mm, response time of less than 1 s, and accuracy of 0.25%) were utilized to measure the flame temperature. They were placed at the nozzle axis and above the nozzle surface at 0.5, 1.5, 4, 7.5, 12, 28, 25 and 33 cm. The method of Brohez et al. [16] was used in correcting the error in the measured temperature. In considering the interference by thermocouples in the flow, the thermocouples were absent during the measurements of flame images, and the temperature was measured separately.

There was no more than 0.3% variation in the inner pressure in the cabin for each case with or without flames. The ambient temperature and humidity in the test section were maintained at 298±1 K and 60%±5%, respectively, in all experimental cases. Each case was repeated three times.

**RESULTS AND DISCUSSION**

**Flame length with Reynolds numbers**

The dependence of $l_F/d$ on Reynolds number shown in Fig. 2 indicates that $l_F/d = 0.123Re$ for the present study, $l_F/d = 0.353Re$ for Yuan et al. [6], and $l_F/d = 0.136Re$ for Sunderland et al. [17]. This selection of coordinate axes is based on common practice in the literature [10, 17, 18]. The best fit obtained from experimental results of the current study is similar to that of Sunderland et al.

![Figure 2. Dependence of dimensionless flame length on the fuel jet exit Reynolds number.](image)

The flame length by Yuan et al. is chosen as the outer limit of the luminous flame front, while ours is determined by the vertical distance above the nozzle exit surface where the occurrence probability is
Furthermore, in Yuan’s study, the soot loading decreased with pressure reduction, and the soot distributed itself through all parts of the flame, and was not just concentrated at the flame tip zone. In the present study, for relative low pressure cases, the soot mainly concentrated itself at the flame tip zone; and for relative high pressure cases, the soot distributed itself at the upper portion of the flame, while the part near the burner was blue. Therefore, the data by Yuan et al. is higher than Sunderland’s and the present study.

To further explore and illustrate the effect of pressure level on flame length, Fig. 3 plots the flame length normalized as $l_F/d/Re$ versus ambient pressure for the experimental results shown in Fig. 2. Based on the preceding discussion on Eq. (3) and consistency seen in Fig. 2, any statistical variation in $l_F/d/Re$ with ambient pressure should quantify the dependence of flame length on pressure level.

![Figure 3.](image)

Analyses show that the value of $l_F/d/Re$ increases with the reduction of ambient pressure according to $l_F/d/Re = 0.12p_\infty^{-0.22}$. Note that the rate of air needed for combustion is proportional to the stoichiometric requirements for combustion [19]. Thus, although the ambient pressure varies, the rate of air consumption remains stable as long as the fuel mass flow rate is fixed. The lower oxygen densities at lower ambient pressures make the flame sheet move upward to obtain adequate fresh air; hence the flame length (or $l_F/d/Re$) slightly increases when the ambient pressure decreases. But this increase of flame length was very small relative to the scale of the flame length.

### Flame length and radius with Froude numbers

$Fr$ and $(l_F/r)(r_F/r)^{4/3}$ were used as axes to plot the experiments as shown in Fig. 4. The exponent of the present experiment result, $(l_F/r)(r_F/r)^{4/3} = 16.55Fr^{0.54}$, is slight larger than the prediction of Eq. (7). This result is probably due to the fact that the process of deriving the flame radius formula neglects the effect of radial convection, through which the buoyancy shrinks flame widths [10]. Some experimental research found that the flame length varies slightly with the changes of ambient pressure, which should be mainly caused by the effects of buoyancy [6, 7].

The relationship of the dimensionless flame length and $Fr$ is plotted in Fig. 5. It was observed that there was a positive correlation between $l_F/d$ and $Fr$, as

$$l_F/d = 20.2Fr^{0.322},$$

(8)
CONCLUSIONS

Laminar buoyant diffusion methane flames were produced by a circular burner with inner diameter of 12 mm in ambient pressures of 45-100 kPa. The fuel mass flow rates varied in the range of 6.0-20.9 mg/s, with jet exit Reynolds numbers of 58-202 and Froude numbers of 0.046-2.8. Major findings are listed below:

(1) The relationship of the dimensionless flame length and Reynolds number was \( \frac{l_F}{d} = 0.123 Re \). Furthermore, the flame length normalized as \( \frac{l_F}{d/Re} \) increased with the reduction of ambient pressure according to \( \frac{l_F}{d/Re} = 0.12 Re^{0.22} \). But this increase of flame length was very small relative to the scale of the flame length.

(2) The dimensionless flame length and radius were analyzed to have the relation of \( (l_F/r)(r_F/r)^{4/3} \sim Fr^{1/3} \), which was in good agreement with the experimental results. The relationship of the dimensionless flame length and Froude number was \( l_F/d = 20.2 Fr^{0.322} \).

REFERENCES